

# A New Open Economy Model With Calvo Pricing

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*Objective: Consider Obstfeld and Rogoff's two-country model, but with a Cobb-Douglas consumption index and Calvo pricing. What does this cause to the dynamics of the model, and how can we solve the model numerically?*

The Matlab program described in this handout is called `studygr.m`

To run the program, launch Matlab and type  
studygr  
in the Command Window. Matlab will then perform all the calculations and display impulse response functions.

## 0.1 Summary of the Main Features:

- aggregate consumption for the two countries is given by the Cobb-Douglas indexes
- all the varieties (monopolistic competition) are aggregated according to a CES consumption index
- the Home country is populated by a continuum of producers/consumers on the interval  $[0, n]$ , while the Foreign country consists of agents on the remaining  $(n, 1]$
- real interest rate is equal for the two countries and is given by the

Fisher relationship

- PPP holds
- Calvo pricing mechanism
- exogenous growth rates of nominal money in the two countries
- government expenditure is equal to zero

## 1 The Model

The Home country is populated by a continuum of producers/consumers on the interval  $[0, n]$ , while the Foreign country consists of agents on the remaining  $(n, 1]$ . In the Home country, each producer/consumer solves the following maximization problem:

$$\begin{aligned}
& \max \quad E_t \sum_{j=0}^{\infty} \beta^j \left[ \log(C_{t+j}) + \frac{\chi}{1-\varepsilon} \left( \frac{M_{t+j}}{P_{t+j}} \right)^{1-\varepsilon} - \frac{\kappa}{2} y_{t+j}(z)^2 \right] \\
& \text{s.t. } P_t F_t + M_t = P_t (1 + r_{t-1}) F_{t-1} + \\
& M_{t-1} \\
& + p_t(z) y_t(z) - P_t C_t - P_t T_t
\end{aligned}$$

An analogous maximization problem is solved by Foreign agents. There is an integrated world capital market in which both countries can borrow and lend. Each agent consumes all the goods produced in the two country, and supplies one differentiated good.

Aggregate consumption for the two countries is given by the Cobb-Douglas indexes:

$$C_t = \frac{(C_{H,t})^n (C_{F,t})^{1-n}}{n^n (1-n)^{1-n}} \text{ for the Home}$$

country

$C_t^* = \frac{(C_{H,t}^*)^n (C_{F,t}^*)^{1-n}}{n^n (1-n)^{1-n}}$  for the Foreign country

However, since differentiated goods are imperfect substitutes in consumption, each individual enjoys a degree of monopolistic competition over the good he produces. All the varieties are aggregated according to a CES consumption index:

$$C_{H,t} = \left[ \left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n c_{H,t}(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

for Home consumption of Home-produced goods

$$C_{F,t} = \left[ \left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 c_{F,t}(z^*)^{\frac{\theta-1}{\theta}} dz^* \right]^{\frac{\theta}{\theta-1}}$$

for Home consumption of Foreign-produced goods

$$C_{H,t}^* = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\theta}} \int_0^n c_{H,t}^*(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

for Foreign consumption of  
Home-produced goods

$$C_{F,t}^* = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\theta}} \int_n^1 c_{F,t}^*(z^*)^{\frac{\theta-1}{\theta}} dz^* \right]^{\frac{\theta}{\theta-1}}$$

for Foreign consumption of  
Foreign-produced goods

If government expenditure on goods is equal to zero, then demand for differentiated goods is equal to:

$$y_t(z) = \left[ \frac{p_t(z)}{P_{Ht}} \right]^{-\theta} T_t^{1-n} C_t^W$$

$$y_t^*(z^*) = \left[ \frac{p_t^*(z^*)}{P_{Ft}^*} \right]^{-\theta} T_t^{-n} C_t^W$$

Where  $C_t^W$  is the world aggregate consumption and  $T_t$  is the terms of trade:

$$C_t^W = \int_0^n C_t(z) dz + \int_n^1 C_t^*(z^*) dz^*$$

$$T_t = \frac{P_{F,t}}{P_{H,t}}$$

We assume that the governments set seignorage equal to transfers:

$$0 = T_t + \frac{M_t - M_{t-1}}{P_t}$$

$$0 = T_t^* + \frac{M_t^* - M_{t-1}^*}{P_t^*}$$

that the real interest rate is equal for the two countries and is given by the Fisher relationship:

$$1 + i_t = E_t \frac{P_{t+1}}{P_t} (1 + r_t)$$

$$1 + i_t^* = E_t \frac{P_{t+1}^*}{P_t^*} (1 + r_t)$$

and that PPP holds at the level of the individual good:

$$p_t(z) = e_t p_t^*(z)$$

$$p_t(z^*) = e_t p_t^*(z^*)$$

## 2 The Calvo pricing mechanism

This is a pricing mechanism that allows to obtain "smooth" impulse responses after a shock. Assume (instead of one period sticky prices) that monopolistically competitive producers/consumers can adjust their individual price, but only with some probability. That is, in each period there is a probability  $\alpha$  of keeping prices constant and a probability  $1 - \alpha$  of changing prices. This implies that the fraction of agents that change price at time  $t$  is equal to  $1 - \alpha$ .

Therefore, an agent changing his

price at time  $t$  solves the following maximization problem:

$$\max_{p_t(z)} E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left[ \frac{1}{C_{t+j}} \frac{1}{P_{t+j}} \cdot p_t(z) \tilde{y}_{t+j}(z) - \frac{\kappa}{2} \tilde{y}_{t+j}(z)^2 \right]$$

$$\text{s.t. } \tilde{y}_{t+j}(z) = \left[ \frac{p_t(z)}{P_{H,t+j}} \right]^{-\theta} T_{t+j}^{1-n} C_{t+j}$$

Explanation:

- Expected "profits" are discounted for the probability that the decision on the price is still maintained for the future, and the probability that the price chosen at date  $t$  will still apply in the future at date  $t + j$  is  $\alpha^k$
- $\tilde{y}_{t+j}(z) = \left[ \frac{p_t(z)}{P_{H,t+j}} \right]^{-\theta} T_{t+j}^{1-n} C_{t+j}$  is demand of good  $z$  at date  $t + j$  under the assumption that  $p_t(z)$  still applies

- $\frac{1}{C_{t+j}} \frac{1}{P_{t+j}}$  is the marginal utility of nominal income (under the assumption of complete markets, this is equal for everybody)
- $\frac{\kappa}{2} \tilde{y}_{t+j}(z)^2$  is the disutility of producing output  $\tilde{y}_{t+j}(z)$

The first order condition is:

$$E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left[ \frac{1}{C_{t+j}} \frac{\tilde{y}_{t+j}(z)}{P_{t+j}} + \frac{\theta}{1-\theta} \kappa \frac{\tilde{y}_{t+j}(z)^2}{p_t(z)} \right] = 0$$

Re-arranging:

$$E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left[ \frac{1}{C_{t+j}} \frac{p_t(z)}{P_{H,t+j}} T_{t+j}^{n-1} \tilde{y}_{t+j}(z) + \frac{\theta}{1-\theta} \kappa \tilde{y}_{t+j}(z)^2 \right] = 0$$

Denoting the relative price by:

$X_{t+j} \equiv \frac{p_t(z)}{P_{H,t+j}}$ , and log-linearizing around the steady state we obtain:

$$E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left[ \begin{array}{c} -\hat{C}_{t+j} + \hat{X}_{t+j} \\ - (1-n) \hat{T}_{t+j} - \hat{y}_{t+j}(z) \end{array} \right] =$$

0

where all hats denote percentage deviations from steady state values.

Notice that Home inflation is given by:  $\pi_{t+1} \equiv \widehat{X}_t - \widehat{X}_{t+1}$

By making use of the following result:

$$\widehat{X}_{t+j} = \frac{\alpha}{1-\alpha} \pi_t - \sum_{s=1}^j \pi_{t+s}$$

and by substituting  $\widehat{y}_{t+j}(z)$  out, and applying the law of iterated expectations, we obtain:

$$\pi_t = \beta E_t \pi_{t+1} + (1 - \alpha\beta) \frac{1-\alpha}{\alpha} \left[ \frac{2}{1+\theta} \widehat{C}_t + \frac{2}{1+\theta} (1 - n) \widehat{T}_t \right]$$

→ the forward-looking Phillips curve for the Home country.

An increase in demand generates an increase in output and, for a given  $E_t \pi_{t+1}$ , an increase in inflation.

Note that, according to this Phillips curve, inflation is a forward looking variable, and that by stabilizing demand (and output) the Central Bank is also stabilizing inflation.

By solving the analogous problem for the Foreign country, we find the following Phillips curve:

$$\pi_t^* = \beta E_t \pi_{t+1}^* + (1 - \alpha\beta) \frac{1-\alpha}{\alpha} \left[ \frac{2}{1+\theta} \widehat{C}_t^* - \frac{2}{1+\theta} n \widehat{T}_t \right]$$

### 3 Monetary Policy

Since we have a Phillips curve written in terms of inflation, it makes sense to specify monetary policy in terms of growth rates and not in terms of money stocks. We can assume, for example, that the growth

rates of nominal money in the two countries follow two exogenous processes,  $\mu_t$  and  $\mu_t^*$ :

$$\frac{M_t}{M_{t-1}} = \mu_t$$

$$\frac{M_t^*}{M_{t-1}^*} = \mu_t^*$$

In order to introduce some persistence in monetary policy, we can assume that the logarithms of  $\mu_t$  and  $\mu_t^*$  are AR(1):

$$\hat{\mu}_t = \rho \hat{\mu}_{t-1} + \epsilon_t$$

$$\hat{\mu}_t^* = \rho^* \hat{\mu}_{t-1}^* + \epsilon_t^*$$

where  $\hat{\mu}_t = \log(\mu_t)$ ,  $\hat{\mu}_t^* = \log(\mu_t^*)$ .

## 4 Equations Listing and Log-linearisation

We first log-linearise all the equations in the model around a steady state with no debt ( $\bar{F}_0 = \bar{F}_0^* = 0$ ). The Cobb-Douglas consumption index then ensures that  $F_t = 0$  for each  $t$ . Without this result, we will have permanent wealth redistributions, and Home and Foreign will not go back to the same initial steady state after a shock, impairing the accuracy of approximations around the initial steady state in the long run.

In this section, we denote with "hats" log-linearised variables, or variables in percentage deviations.

Since we cannot log-linearise around  $\bar{F}_0 = 0$ , we linearise with respect to  $F_t$ , defining  $dF_t = F_t - \bar{F}_0$ .

### 4.0.1 Aggregate resource constraint

By adding the Home and Foreign budget constraints, and using  $ndF_t + (1 - n)dF_t^* = 0$ ,  $\bar{C}_0 = \bar{C}_0^*$ , we get:

$$dF_t = \frac{1}{\beta}dF_{t-1} - (1 - n)\bar{C}_0 \left( \hat{C}_t - \hat{C}_t^* \right)$$

### 4.0.2 Euler Equations

By log-linearising the Home Euler equation

$$E_t C_{t+1} = \beta (1 + r_t) C_t$$

we get:

$$E_t \hat{C}_{t+1} - \hat{C}_t = (1 - \beta) \hat{r}_t$$

Analogously, for the Foreign country:

$$E_t \widehat{C}_{t+1}^* - \widehat{C}_t^* = (1 - \beta) \widehat{r}_t$$

### 4.0.3 LM curve

The money market equilibrium condition for Home is analogous to the one in Obstfeld and Rogoff's Redux model:

$$\frac{M_t}{P_t} = m_t = \left[ \chi C_t \left( \frac{1+i_t}{i_t} \right) \right]^{\frac{1}{\varepsilon}}$$

which log-linearised becomes:

$$\widehat{m}_t = \frac{1}{\varepsilon} \widehat{C}_t - \frac{\beta}{\varepsilon} \widehat{i}_t$$

Analogously, for the Foreign country:

$$\widehat{m}_t^* = \frac{1}{\varepsilon} \widehat{C}_t^* - \frac{\beta}{\varepsilon} \widehat{i}_t^*$$

### 4.0.4 Fisher parity conditions

Using the fact that the growth in the price level depends on both inflation and exchange rate movements:

$$E_t \widehat{P}_{t+1} - \widehat{P}_t = n E_t \pi_{t+1} +$$

$$(1 - n) E_t \pi_{t+1}^* +$$

$$(1 - n) (E_t \hat{\varepsilon}_{t+1} - \hat{\varepsilon}_t)$$

we get

$$\hat{r}_t \hat{i}_t = \left( \frac{1}{1-\beta} \right) \left( \begin{array}{l} n E_t \pi_{t+1} + (1 - n) E_t \pi_{t+1}^* \\ + (1 - n) (E_t \hat{\varepsilon}_{t+1} - \hat{\varepsilon}_t) \end{array} \right) +$$

for the Home country, and

$$\hat{r}_t \hat{i}_t^* = \left( \frac{1}{1-\beta} \right) \left( \begin{array}{l} n E_t \pi_{t+1} + (1 - n) E_t \pi_{t+1}^* \\ - n (E_t \hat{\varepsilon}_{t+1} - \hat{\varepsilon}_t) \end{array} \right) +$$

for the Foreign country.

#### 4.0.5 Money creation

Since we allow for positive money growth and inflation, we normalize all the equations so that prices and nominal money stocks (that are not stationary) disappear from the system, and we can work with only

stationary variables (real money balances and inflation rates).

Basically, we make the following substitution:

$$\widehat{m}_t = \widehat{M}_t - \widehat{P}_t$$

and using  $\frac{M_t}{M_{t-1}} = \mu_t$  we can show for the Home country that:

$$\widehat{m}_t - \widehat{m}_{t-1} + n\pi_t + (1 - n)\pi_t^* + (1 - n)(\widehat{\varepsilon}_t - \widehat{\varepsilon}_{t-1}) = \widehat{\mu}_t$$

For the Foreign country:

$$\widehat{m}_t^* - \widehat{m}_{t-1}^* + n\pi_t + (1 - n)\pi_t^* - n(\widehat{\varepsilon}_t - \widehat{\varepsilon}_{t-1}) = \widehat{\mu}_t^*$$

#### **4.0.6 The law of motion for the terms of trade**

We need a dynamic relationship between the terms of trade, inflation and the exchange rate. Starting from the definition  $T_t = \frac{P_{F,t}}{P_{H,t}}$ , we can show

that:

$$\widehat{T}_t = \widehat{T}_{t-1} + (\widehat{\varepsilon}_t - \widehat{\varepsilon}_{t-1}) - \pi_t + \pi_t^*$$

## 5 The solution algorithm

The system of equations that we want to solve is:

1.  $dF_t = \frac{1}{\beta}dF_{t-1} - (1 - n)\bar{C}_0\widehat{C}_t + (1 - n)\bar{C}_0\widehat{C}_t^*$
2.  $E_t\widehat{C}_{t+1} - (1 - \beta)\widehat{r}_t = \widehat{C}_t$
3.  $E_t\widehat{C}_{t+1}^* - (1 - \beta)\widehat{r}_t = \widehat{C}_t^*$
4.  $\widehat{m}_t = \frac{1}{\varepsilon}\widehat{C}_t - \frac{\beta}{\varepsilon}\widehat{i}_t$
5.  $\widehat{m}_t^* = \frac{1}{\varepsilon}\widehat{C}_t^* - \frac{\beta}{\varepsilon}\widehat{i}_t^*$
6.  $nE_t\pi_{t+1} + (1 - n)E_t\pi_{t+1}^* + (1 - \beta)\widehat{r}_t - (1 - n)\widehat{\varepsilon}_t + (1 - n)E_t\widehat{x}_{t+1} = (1 - \beta)\widehat{i}_t$
7.  $nE_t\pi_{t+1} + (1 - n)E_t\pi_{t+1}^* + (1 - \beta)\widehat{r}_t + n\widehat{\varepsilon}_t - nE_t\widehat{x}_{t+1} = (1 - \beta)\widehat{i}_t^*$

$$\mathbf{8.} \quad \widehat{m}_t + (1 - n)\widehat{\varepsilon}_t = \widehat{m}_{t-1} - n\pi_t - (1 - n)\pi_t^*$$

$$+ (1 - n)\widehat{\varepsilon}_{t-1} + \widehat{\mu}_t$$

$$\mathbf{9.} \quad \widehat{m}_t^* - n\widehat{\varepsilon}_t = \widehat{m}_{t-1}^* - n\pi_t - (1 - n)\pi_t^*$$

$$- n\widehat{\varepsilon}_{t-1} + \widehat{\mu}_t^*$$

$$\mathbf{10.} \quad \widehat{\varepsilon}_t = \widehat{x}_t$$

$$\mathbf{11.} \quad -\frac{\alpha}{1-\alpha}\frac{1}{1-\alpha\beta}\beta E_t\pi_{t+1} - \frac{2}{1+\theta}(1-n)\widehat{T}_t = \frac{2}{1+\theta}\widehat{C}_t$$

$$- \frac{\alpha}{1-\alpha}\frac{1}{1-\alpha\beta}\pi_t$$

$$\mathbf{12.} \quad -\frac{\alpha}{1-\alpha}\frac{1}{1-\alpha\beta}\beta E_t\pi_{t+1}^* + \frac{2}{1+\theta}n\widehat{T}_t = \frac{2}{1+\theta}\widehat{C}_t^*$$

$$- \frac{\alpha}{1-\alpha}\frac{1}{1-\alpha\beta}\pi_t^*$$

$$\mathbf{13.} \quad \widehat{T}_t - \widehat{\varepsilon}_t = -\pi_t + \pi_t^* + \widehat{T}_{t-1} - \widehat{\varepsilon}_{t-1}$$

$$\mathbf{14.} \quad E_t\widehat{\mu}_{t+1} = \rho\widehat{\mu}_t$$

$$\mathbf{15.} \quad E_t\widehat{\mu}_{t+1}^* = \rho^*\widehat{\mu}_t^*$$

Where we have substituted  $\widehat{\varepsilon}_{t+1}$

with  $\hat{x}_{t+1}$ . This system can be represented in the following form:

$$PSI * y_{t+1} = PHI * y_t$$

where  $y_t$  is a column vector with the variables  $dF_{t-1}$ ,  $\hat{C}_t$ ,  $\hat{C}_t^*$ ,  $\hat{m}_{t-1}$ ,  $\hat{m}_{t-1}^*$ ,  $\pi_t$ ,  $\pi_t^*$ ,  $\hat{i}_t$ ,  $\hat{i}_t^*$ ,  $\hat{r}_{t-1}$ ,  $\hat{\varepsilon}_{t-1}$ ,  $\hat{x}_t$ ,  $\hat{T}_{t-1}$ ,  $\hat{\mu}_t$ ,  $\hat{\mu}_t^*$ . Of these variables, 8 are known at time  $t$ :  $dF_{t-1}$ ,  $\hat{m}_{t-1}$ ,  $\hat{m}_{t-1}^*$ ,  $\hat{r}_{t-1}$ ,  $\hat{\varepsilon}_{t-1}$ ,  $\hat{T}_{t-1}$ ,  $\hat{\mu}_t$ ,  $\hat{\mu}_t^*$ , and 7 are decided at time  $t$ :  $\hat{C}_t$ ,  $\hat{C}_t^*$ ,  $\pi_t$ ,  $\pi_t^*$ ,  $\hat{i}_t$ ,  $\hat{i}_t^*$ ,  $\hat{x}_t$  ( $\hat{x}_t$  is equal to  $\hat{\varepsilon}_t$ ).

We can then apply a solution technique which is essentially the same as the one described in Judd's book, pages 339-340, or in Blanchard and Khan's paper. These algorithms make use of a theorem which says that  $PSI * y_{t+1} = PHI * y_t$  is

equivalent to the system:

$$VV * y_{t+1} = DD * VV * y_t$$

where  $DD$  is the matrix of (generalised) eigenvalues and  $VV^{-1}$  is the matrix of (generalised) eigenvectors of  $PSI$  and  $PHI$ . We can have a unique solution only when the number of eigenvalues greater than one in modulus is equal to the number of free variables (7), and the solution technique amounts to setting the equations in  $VV * y_t$  which correspond to those eigenvalues equal to zero.

This technique can be summarised in four steps.

Step 1: find the rows in  $DD$  with eigenvalues greater than one in

modulus.

Step 2: construct a submatrix  $\overline{VV}$  which is derived from  $VV$  but has only those rows, write  $\overline{VV} * y_t = 0$  (our  $\overline{VV}$  has 7 rows).

Step 3: re-write  $\overline{VV} * y_t = 0$  separating free and predetermined variables:

$$\overline{VV}y_t = AA * \begin{bmatrix} \widehat{C}_t \\ \widehat{C}_t^* \\ \pi_t \\ \pi_t^* \\ \widehat{i}_t \\ \widehat{i}_t^* \\ \widehat{x}_t \end{bmatrix} + BB *$$

$$\begin{bmatrix} dF_{t-1} \\ \widehat{m}_{t-1} \\ \widehat{m}_{t-1}^* \\ \widehat{r}_{t-1} \\ \widehat{\varepsilon}_{t-1} \\ \widehat{T}_{t-1} \\ \widehat{\mu}_t \\ \widehat{\mu}_t^* \end{bmatrix} = 0$$

Step 4: the solution is represented by a matrix  $PP = -AA^{-1} * BB$  such that

$$\begin{bmatrix} \widehat{C}_t \\ \widehat{C}_t^* \\ \pi_t \\ \pi_t^* \\ \widehat{i}_t \\ \widehat{i}_t^* \\ \widehat{x}_t \end{bmatrix} = PP^* \begin{bmatrix} dF_{t-1} \\ \widehat{m}_{t-1} \\ \widehat{m}_{t-1}^* \\ \widehat{r}_{t-1} \\ \widehat{\varepsilon}_{t-1} \\ \widehat{T}_{t-1} \\ \widehat{\mu}_t \\ \widehat{\mu}_t^* \end{bmatrix}$$

While the coefficients in matrix  $PP$  stay constant, the solution path for all endogenous variables is updated period by period by agents, after observing the shocks  $\widehat{\mu}_t$  and  $\widehat{\mu}_t^*$ .

**Remark:** what is the analogy between this  $\infty$ -horizon solution method and the one used by Obstfeld and Rogoff in their Redux model?

In OR's model price rigidity

”lasts” for only one period, therefore there are only 3 ”dates” to care about: the initial steady state (say time 0), the short run (say time  $t$ ), and the long run (say  $t + 1$ ). The  $t + 1$  system has one degree of freedom:  $F_{t+1}$ , assets in the ”new” steady state is unknown. However, if we impose the stability condition that debt cannot increase in each period to finance consumption, which amounts to  $F_{t+1} = F_t$ , so whatever is carried from period  $t$  becomes the new steady state level of  $F$ , we can find the whole solution path. In their book, O&R first find the long-run steady state for a given wealth redistribution, and then they

find the short-run dynamics which satisfy the stability condition.