Welfare and Macroeconomic Interdependence

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References:

- NBER working paper 6307
- QJE, 116(2), May 2001.

Outline:

1. The Model
2. Solving the Model
3. Results
   ”**But ... what for**?”
4. Welfare Analysis

1. The Model

Main features:

- general equilibrium model;
- monopolistic competition: the world is populated by a continuum of agents, each agent is a monopolistic supplier of a (specific) labour input;
- nominal rigidities: nominal wages in period t are predetermined with contracts signed at time $t-1$;
- two countries, Home and Foreign, each specialized in the production of a traded good.

1.1 Preferences

In each country there is a continuum of agents, with population size normalized to 1. The lifetime utility of Home agent $j \in [0,1]$ is given by:

$$U_t(j) = \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\begin{array}{c} \frac{C_\tau(j)^{1-\rho}}{1-\rho} + \chi \ln \frac{M_\tau(j)}{P_\tau} \\ +V(G_\tau) - \frac{\kappa}{2} l_\tau(j)^2 \end{array} \right]$$

where:

G: government expenditure

l: labour hours

The lifetime utility of the Foreign agent $j^* \in [0,1]$ is similarly defined.

1.2 Consumption and price indexes

The consumption indexes for the Home agent j and the Foreign agent j^* are defined as:

Home: $C_t(j) \equiv (C_{H,t}(j))^{\gamma} (C_{F,t}(j))^{1-\gamma}$

Foreign: $C_t^*(j^*) \equiv (C_{H,t}^*(j^*))^{\gamma} (C_{F,t}^*(j^*))^{1-\gamma}$

The corresponding price indexes are then:

Home: $P_t \equiv \frac{1}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} (P_{H,t})^{\gamma} (E_t P_{F,t}^*)^{1-\gamma}$

Foreign: $P_t^* \equiv \frac{1}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \left(\frac{P_{H,t}}{E_t}\right)^{\gamma} (P_{F,t}^*)^{1-\gamma}$

where:

E: nominal exchange rate

P_H is the price of the Home good in domestic currency, P_F^* is the price of the Foreign good in local (foreign) currency. The price index is found as the solution to a minimization problem: it is the minimum expenditure required to buy one unit of the composite good C, given P_H and P_F.

1.3 Technology and Production

Output is produced with a continuum of differentiated labour inputs provided by domestic agents:

$$Y_t = \left(\int_0^1 l_t(j)^{\frac{\phi-1}{\phi}} dj\right)^{\frac{\phi}{\phi-1}}$$

The labour demand for each type of labour is derived from the firms' profit maximization problem:

$$\max_{l_t(j)} \quad P_{H,t}Y_t - \int_0^1 w_t(j)l_t(j)dj$$

$$\text{s.t.} \quad Y_t = \left(\int_0^1 l_t(j)^{\frac{\phi-1}{\phi}} dj\right)^{\frac{\phi}{\phi-1}}$$

$$\rightarrow l_t(j) = \left(\frac{w_t(j)}{P_{H,t}}\right)^{-\phi} Y_t$$

1.4 Law of One Price

The law of one price holds: goods prices expressed in the same currency are equal across countries:

$$P_{H,t} = P^*_{H,t}E_t$$

$$P_{F,t} = P^*_{F,t}E_t$$

also note that consumption-based PPP holds: $P_t = E_t P^*_t$.

We can therefore see that the model proposed by Corsetti and Pesenti (CP) is essentially analogous to the benchmark open-economy model by Obstfeld and Rogoff (OR). We can think of CP's model as OR's modified ad follows:

1. labour markets are imperfectly competitive (each agent is a monopolistic supplier of her own labour input), while goods markets are competitive (the opposite in OR);
2. public goods enter the individuals' utility function, and governments spend on the goods of their own country only;
3. countries are of equal size;
4. the elasticity of substitution between foreign and domestic goods (given by the consumption index) is equal to 1, while the "degree of monopolistic competition" is $\frac{1}{\phi}$ (in OR the same parameter plays the double role of

elasticity of substitution between foreign and domestic goods and index of monopolistic distortion).

Only the last point is essential to CP's results: the same conclusions about the impact of monetary policy could be derived without 1, 2 and 3. However, 2 plays certainly a role into the transmission mechanism, and welfare effects, of fiscal policy.

1.5 The Uncovered Interest Parity Condition

In CP real interest rates are set equal across countries by arbitrage. By definition:

$$(1 + i_t)\frac{P_t}{P_{t+1}} \equiv 1 + r_t$$

$$(1 + i_t^*)\frac{P_t^*}{P_{t+1}^*} \equiv 1 + r_t$$

The two equations above, together with PPP, imply:

$$1 + i_t = \frac{E_{t+1}}{E_t}(1 + i_t^*)$$

that is, the UIP condition always holds in the model.

2. Solving the Model

The Home agent $j \in [0,1]$ solves the following maximization problem:

$$\max \sum_{\tau=t}^{\infty} \beta^{\tau-t}\left[\begin{array}{c} \frac{C_\tau(j)^{1-\rho}}{1-\rho} + \chi \ln \frac{M_\tau(j)}{P_\tau} \\ +V(G_\tau) - \frac{\kappa}{2} l_\tau(j)^2 \end{array}\right]$$

s.t. $B_t(j) + M_t(j) = (1 + i_{t-1})B_{t-1}(j) + M_{t-1}(j)$

$+w_t(j)l_t(j) - P_tT_t(j) - P_{H,t}C_{H,t}(j) - P_{F,t}C_{F,t}(j)$

$l_t(j) = \left(\frac{w_t(j)}{P_{H,t}}\right)^{-\phi} Y_t$

→! agent takes into account firm's labour demand

B: international bond

i: nominal interest rate

T: government transfers

The Foreign agent $j^* \in [0,1]$ solves:

$$\max \sum_{\tau=t}^{\infty} \beta^{\tau-t}\left[\begin{array}{c} \frac{C_\tau^*(j^*)^{1-\rho}}{1-\rho} + \chi \ln \frac{M_\tau^*(j^*)}{P_\tau^*} \\ +V(G_\tau^*) - \frac{\kappa}{2} l_\tau^*(j^*)^2 \end{array}\right]$$

s.t.

$\frac{B_t(j^*)}{E_t} + M_t^*(j^*) = (1 + i_{t-1})\frac{B_{t-1}(j^*)}{E_t} + M_{t-1}^*(j^*) +$

$w_t^*(j^*)l_t^*(j^*) - P_t^* T_t^*(j^*) - P_{H,t}^* C_{H,t}^*(j^*)$
$-P_{F,t}^* C_{F,t}^*(j^*)$

$l_t^*(j^*) = \left(\frac{w_t^*(j^*)}{P_{H,t}^*}\right)^{-\phi^*} Y_t^*$

Note: here the timing convention differs from the one adopted by CP and is the same as in OR. Both M_{t-1} and B_{t-1} denote stocks cumulated during period $t-1$ and carried over into period t. This modification allows us to recover the first-order conditions (3) and (4) as they are written in CP's two papers.

2.1 The system of equations

The model is constituted by a system of equations, coming from the first-order conditions from the Home and Foreign maximization problems, plus all the other constraints and definitions. Money and government expenditure are exogenous.

First-order conditions:

(1) $C_t(j)^{-\rho} = \beta(1+r_t)C_{t+1}(j)^{-\rho}$

(2) $C_t^*(j^*)^{-\rho} = \beta(1+r_t)C_{t+1}^*(j^*)^{-\rho}$

(3) $\frac{M_t(j)}{P_t} = \chi \frac{1+i_t}{i_t} C_t(j)^{\rho}$

(4) $\frac{M_t^*(j^*)}{P_t^*} = \chi^* \frac{1+i_t^*}{i_t^*} C_t(j^*)^{\rho}$

(5) $l_t(j) = \frac{\phi-1}{\kappa\phi} \frac{w_t(j)}{P_t} C_t(j)^{-\rho}$

(6) $l_t^*(j^*) = \frac{\phi^*-1}{\kappa^*\phi^*} \frac{w_t^*(j^*)}{P_t^*} C_t^*(j^*)^{-\rho}$

Individual budget constraints:

(7) $B_t(j) + M_t(j) = (1+i_{t-1})B_{t-1}(j) + M_{t-1}(j) +$
$w_t(j)l_t(j) - P_t T_t(j) - P_{H,t}C_{H,t}(j) - P_{F,t}C_{F,t}(j)$

(8)
$\frac{B_t^*(j^*)}{E_t} + M_t^*(j^*) = (1+i_{t-1})\frac{B_{t-1}^*(j^*)}{E_t} + M_{t-1}^*(j^*) +$
$w_t^*(j^*)l_t^*(j^*) - P_t^* T_t^*(j^*) - P_{H,t}^* C_{H,t}^*(j^*)$
$-P_{F,t}^* C_{F,t}^*(j^*)$

Government budget constraint in Home and Foreign (Money demand is always equal to money supply):

(9) $\int_0^1 M_t(j)dj - \int_0^1 M_{t-1}(j)dj + P_t \int_0^1 T_t(j)dj$
$= P_{H,t}G_t$

(10)
$\int_0^1 M_t^*(j^*)dj^* - \int_0^1 M_{t-1}^*(j^*)dj^* + P_t^* \int_0^1 T_t^*(j^*)dj^*$
$= P_{F,t}^* G_t^*$

Firms' labour demand:

(11) $l_t(j) = \left(\frac{w_t(j)}{P_{H,t}}\right)^{-\phi} Y_t$

(12) $l_t^*(j^*) = \left(\frac{w_t^*(j^*)}{P_{F,t}^*}\right)^{-\phi^*} Y_t^*$

Definitions:

(13) $P_t \equiv \frac{1}{\gamma^\gamma(1-\gamma)^{1-\gamma}}(P_{H,t})^\gamma (E_t P_{F,t}^*)^{1-\gamma}$

(14) $P_t^* \equiv \frac{1}{\gamma^\gamma(1-\gamma)^{1-\gamma}}\left(\frac{P_{H,t}}{E_t}\right)^\gamma (P_{F,t}^*)^{1-\gamma}$

Law of one price:

(15) $P_{H,t} = P_{H,t}^* E_t$

(16) $P_{F,t} = P_{F,t}^* E_t$

Zero net supply of international bond:

(17) $\int_0^1 B_t(j)dj + \int_0^1 B_t^*(j^*)dj^* = 0$

Production:

(18) $Y_t = \left(\int_0^1 l_t(j)^{\frac{\phi-1}{\phi}} dj\right)^{\frac{\phi}{\phi-1}}$

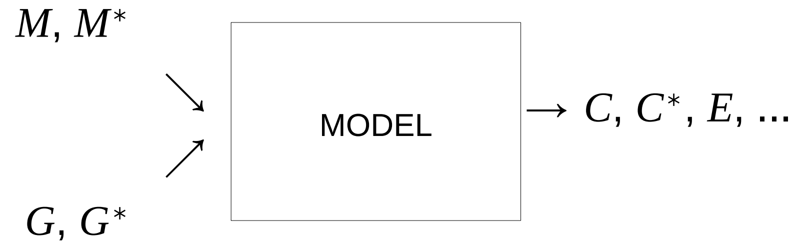
(19) $Y_t^* = \left(\int_0^1 l_t^*(j^*)^{\frac{\phi^*-1}{\phi^*}} dj^*\right)^{\frac{\phi^*}{\phi^*-1}}$

Worldwide resource constraint:

(20) $Y_t = G_t + \int_0^1 C_{H,t}(j)dj + \int_0^1 C_{H,t}^*(j^*)dj^*$

(21) $Y_t^* = G_t^* + \int_0^1 C_{F,t}(j)dj + \int_0^1 C_{F,t}^*(j^*)dj^*$

M, M^*, G and G^* are policy instruments.



The system is simplified by considering an equilibrium in which agents are symmetric within each country, therefore dropping all the indexes j and j^* and interpreting all the variables in per-capita terms.

That is:

$l_t(j) = l_t$

$l_t^*(j^*) = l_t^*$

As a consequence:

$Y_t = l_t$

$Y_t^* = l_t^*$

$P_{H,t} = w_t$

$P_{F,t}^* = w_t^*$

After all these simplifications (integrals disappear because all agents now are equal within each country), and remembering that the Cobb-Douglas consumption indexes ensure us that:

$$C_{H,t} = \gamma \frac{P_t C_t}{P_{H,t}}$$

$$C^*_{H,t} = \gamma \frac{P^*_t C^*_t}{P^*_{H,t}}$$

$$C_{F,t} = (1-\gamma)\frac{P_t C_t}{P_{F,t}}$$

$$C^*_{F,t} = (1-\gamma)\frac{P^*_t C^*_t}{P^*_{F,t}}$$

the system becomes much simpler, so we are left with the following equations:

(22) $C_t^{-\rho} = \beta(1+r_t)C_{t+1}^{-\rho}$

(23) $C_t^{*-\rho} = \beta(1+r_t)C_{t+1}^{*-\rho}$

(24) $\frac{M_t}{P_t} = \chi \frac{1+i_t}{i_t} C_t^{\rho}$

(25) $\frac{M^*_t}{P^*_t} = \chi^* \frac{1+i^*_t}{i^*_t} C_t^{*\rho}$

(26) $Y_t = \frac{\phi-1}{\kappa\phi}\frac{P_{H,t}}{P_t} C_t^{-\rho}$

(27) $Y^*_t = \frac{\phi^*-1}{\kappa^*\phi^*}\frac{P^*_{F,t}}{P^*_t} C_t^{*-\rho}$

(28) $(1+i_t)\frac{P_t}{P_{t+1}} \equiv 1+r_t$

(29) $(1+i^*_t)\frac{P^*_t}{P^*_{t+1}} \equiv 1+r_t$

(30) $B_t = (1+i_{t-1})B_{t-1} + P_{H,t}\frac{Y_t}{g} - P_t C_t$

(31) $\frac{B^*_t}{E_t} = (1+i_{t-1})\frac{B^*_{t-1}}{E_t} + P^*_{F,t}\frac{Y^*_t}{g^*} - P^*_t C^*_t$

(32) $\frac{P_{H,t}}{P_t}\frac{Y_t}{g} = \gamma(C_t + C^*_t)$

(33) $\frac{P^*_{F,t}}{P^*_t}\frac{Y^*_t}{g^*} = (1-\gamma)(C_t + C^*_t)$

$$g = \frac{Y_t}{Y_t - G_t}$$

$$g^* = \frac{Y^*_t}{Y^*_t - G^*_t}$$

! CP's model is entirely represented by equations (22)-(33), which correspond to the equations in Table 1 of the QJE paper.

What is the role of monopolistic competition? Technically, monopolistic competition is responsible only for introducing a wedge (of size $\frac{\phi}{\phi-1}$, $\frac{\phi^*}{\phi^*-1}$) between marginal disutility and wages in the labour-leisure tradeoffs (26) and (27). However, as it is explained later, these two equations do not play any role in the short run. In the short-run, output is demand-determined, where demand means equations (32) and (33). Without monopolistic competition, the parameters ϕ, ϕ^* would not enter equations (26) and (27), but the rest of the model would be unchanged, and so its predictions, since ϕ

& ϕ^* are constant. Monopolistic competition is more of a convenient modelling tool. It is a device to justify the assumption that in the short run output is demand-determined. In fact, since wages are above the marginal disutility of working, it is optimal to accommodate the increase in demand for labour after a shock.

Alternatively, we can say that because prices are fixed in the short run agents ”loose” their monopoly power. Instead of setting the wage they accommodate demand fully, so (26) and (27) disappear in the short run (and reappear in the long run). Also notice that if (26) stayed Home inflation would bring Y_t down after a Home monetary expansion.

Because of the wedge $\frac{\phi}{\phi-1}$ ($\frac{\phi^*}{\phi^*-1}$), with perfect competition the level of output produced will be higher. In order to eliminate this inefficiency (which, as explained above, is not affecting the transmission mechanism anyway), some models introduce a government subsidy so as to bring output at the first best (perfect competition) level.

2.2 The closed-form solution

It is explained later that, contrary to OR, in CP’s model shocks do not lead to a permanent redistribution of assets. This advantage allows CP to further simplify the system of equations (22)-(33), and they are able to obtain a closed form solution, while OR’s model cannot be solved without local approximations.

How to find the closed form solution?

CP consider the effects in the short run and in the long run of a permanent monetary (fiscal) shock. They introduce nominal rigidities in the form of nominal wage contracts: nominal wages in period t are predetermined with contracts signed at time $t-1$. They solve the system three times: for an initial steady state with zero assets, for the short run and for a final long-run steady state. The short run coincides with the length of the wage contracts, and therefore lasts one period only.

In practice:

1. assume that in the initial steady state $B_0 = B_0^* = 0$: variables in the initial (pre-shock) steady state are indexed by the subscript 0;
2. solve the system (22)-(33) for the initial (pre-shock) steady state, for given initial levels of M_0, M_0^*, G_0, G_0^*, that is, find the pre-shock steady state value of all the other variables: C, C^*, l, l^*, ...;
3. an unexpected permanent monetary (or fiscal) shock hits the economy, inducing a deviation from the initial steady state values. Re-write the system in this way: (i) introduce new levels of M, M^* (or G, G^* in the case of a fiscal shock. Variables in the short run are not indexed) (ii) replace equations (26) and (27) by the constraint $w_0 = w$ (which implies $P_{H,0} = P_H$) and $w_0^* = w^*$ (which implies $P_{F,0} = P_F$). Find the short run value of the variables of this system.
4. in the long run, wages are free to adjust: re-write and solve the system, with equations (26) and (27) again, but keeping the same levels (because the shock is permanent) for M, M^* (or G, G^*).

The reason why (26) and (27) do not hold in the short run is the assumption of nominal rigidity. (26) and (27) are derived from the efficiency condition for labour allocation (marginal disutility of working = marginal revenue). But in the short run wages are fixed and workers simply meet firms' labour demand at the given wage. This implies that labour and output are not efficient in the short run.

Technical note: CP make sure that (in the short run, when wages are fixed) $\frac{w_t}{P_t} \geq \kappa l_t C_t^\rho$. This means that they consider only shocks that do not lead to a violation of this constraint.

Table 2: Solution of the model

Determinants of Home welfare

C	$=$	$a_1\ (\tilde{M}_W)^{\frac{1}{\rho}}$	Short-run consumption
Y	$=$	$a_2\ (\tilde{M}_R)^{1-\gamma}\ (\tilde{M}_W)^{\frac{1}{\rho}} + G$	Short-run output
$\tilde{M}/P$	$=$	$a_3\ \tilde{M}_W$	Short-run real balances
$\bar{C}$	$=$	$a_4\ (\bar{g}_W)^{-\frac{1}{1+\rho}}$	Long-run consumption
$\bar{Y}$	$=$	$a_5\ \bar{g}^{\frac{1}{2}}\ (\bar{g}_W)^{-[illegible]}$	Long-run output
$\bar{M}/\bar{P}$	$=$	$a_6\ (\bar{g}_W)^{-\frac{1}{1+\rho}}$	Long-run real balances

Prices

$1+r$	$=$	$a_7\ (\tilde{M}_W)^{-1}\ (\bar{g}_W)^{-\frac{1}{1+\rho}}$	Short-run real interest rate
$\mathcal{E}P_F^*/P_H$	$=$	$a_8\ \tilde{M}_R$	Short-run terms of trade
$\mathcal{E} = \bar{\mathcal{E}}$	$=$	$a_9\ \tilde{M}_R$	Nominal exchange rate
$\bar{\mathcal{E}}P_F^*/\bar{P}_H$	$=$	$a_{10}\ (\bar{g}_R)^{-\frac{1}{2}}$	Long-run terms of trade
$\bar{P}_H$	$=$	$a_{11}\ \bar{M}\ (\bar{g}_W)^{-[illegible]}\ \bar{g}^{[illegible]}$	Long-run Home good price

Notes: The index R refers to ratios of Home to Foreign variables. The index W refers to geometric averages of Home and Foreign variables with weights γ and $1-\gamma$. The constants are defined below, where the subscript 0 indexes pre-shock levels.

$a_1 = $ [illegible]

$a_2 = $ [illegible]

$a_3 = $ [illegible]

$a_4 = $ [illegible]

$a_5 = $ [illegible]

$a_6 = $ [illegible]

$a_7 = $ [illegible]

$a_8 = $ [illegible]

$a_9 = $ [illegible]

$a_{10} = $ [illegible]

$a_{11} = $ [illegible]

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Source: QJE 116(2), May 2001.

2.3 The graphical solution

In their NBER working paper CP also propose a graphical apparatus for long-run and short-run analysis. They derive three equilibrium relations between domestic consumption and domestic output/employment.

The goods market equilibrium condition (GE) is derived from the current account identities (30) and (31), and it implies a positive relationship between long-run consumption and long-run output (variables in the long run or steady state are indexed by upperbars):

GE: $\bar{Y} \propto (\bar{g}^*)^{\frac{1-\gamma}{2}}\, \bar{g}^{\frac{1+\gamma}{2}}\, \bar{C}$

The relationship is positive because when Home agents increase their steady-state consumption, demand for Home goods increases proportionally, increasing output.

The labour market equilibrium condition (LE) is derived from the labour-leisure tradeoffs (26) and (27):

LE: $\bar{Y} \propto \left(\frac{\bar{g}}{\bar{g}^*}\right)^{\frac{1-\gamma}{2}} \bar{C}^{-\rho}$

The relationship is negative because an increase in steady-state consumption makes Home agents more willing to enjoy leisure, thus reducing output.

The money market equilibrium condition (ME) is derived from the money market equilibrium equations (24) and (25):

ME: $\bar{C}^{\rho} \propto \frac{\bar{M}^{\gamma}(\bar{M}^{*})^{1-\gamma}}{(P_H)^{\gamma}(P_F^{*})^{1-\gamma}}$

This equation draws an horizontal line in the $\left(\bar{C},\bar{Y}\right)$ space. The ME locus shifts upwards when real money balances increase.

The GE, LE and ME schedules can be derived analogously for the short run values of C and Y.

CP then proceed as follows:

- Long-run: looking at the equations of the GE and LE schedules, we observe that monetary shocks do not affect $\bar{Y}$ and $\bar{C}$. In the long run, consumption and output are determined exclusively by the intersection of the GE locus with the LE locus. In CP's model money is neutral in the long-run (as it is explained later), and the ME locus only determines the level of good prices.
- Short-run: the short run equilibrium levels of consumption and output level correspond to the intersection point between GE and ME. In the presence of short-run nominal rigidities and monopolistic competition in the labour market, agents do not necessarily operate on their labour supply schedule, so that the LE locus is irrelevant in determining the short-run equilibrium allocations.

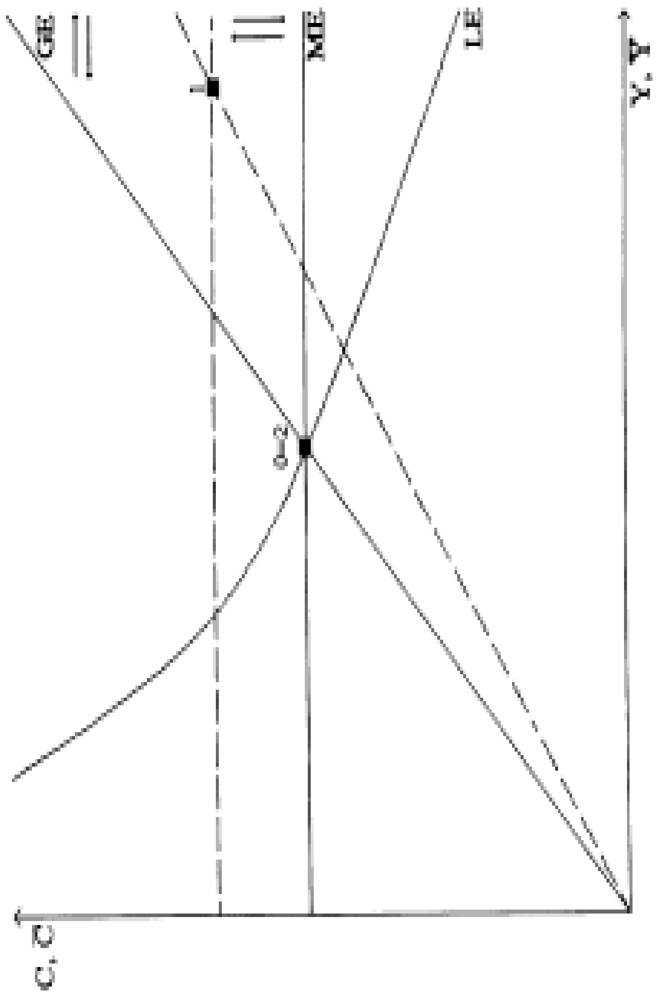


Figure 3. Effects of Home monetary expansions

Source: NBER wp 6307.

Explanation of Figure 3: The economy starts off at point 0, at the intersection between GE, LE and ME.

Short run: A monetary expansion in the Home country shifts the ME locus upward, as the domestic monetary transfer raises real money balances at Home and abroad, requiring higher levels of consumption to maintain equilibrium in the national money market. At the same time, the nominal depreciation of the exchange rate (explained later) reduces the relative price of Home goods: the demand for Home goods increases for any level of Home consumption, and the GE locus tilts downward. The new equilibrium is at point 1, corresponding to higher consumption and output in the Home country. Since prices do not adjust instantaneously to the new fundamentals, the short-run allocation does not lie along the LE curve.

Long run: As the monetary shock does not affect the current account, monetary policy has no long-run effects. In the long-run, world real balances as well as the real exchange rate move back to their original equilibrium levels. Both LE and GE return to their initial positions, so that the long-run equilibrium (point 2) coincides with the original steady state allocation.

3. Results

The main predictions of CP's model can be grouped as follows:

1. Monetary shocks do not affect the current account
2. A permanent monetary expansion is neutral in the long run
3. The exchange rate completely adjusts on impact
4. A permanent fiscal expansion changes the new steady state allocation of employment and consumption

Technical note: results 1) and 2) are employed by CP in the derivation of the analytical and closed-form solutions.

The following subsections are devoted first to the explanation of the transmission mechanism of monetary policy, and then to the explanation of the above results.

3.1 The transmission of monetary shocks

CP re-do the classical Dornbusch exercise of an unanticipated permanent increase in Home money supply, occurring at time t.

$$C_H \uparrow$$

$$\nearrow$$

$$M \uparrow \quad \overset{\text{Home wealth}\uparrow}{\longrightarrow} \quad C \uparrow$$

$$\searrow$$

$$C_F \uparrow \longrightarrow E \uparrow \longrightarrow C_H \uparrow$$

A permanent monetary expansion in the Home country raises Home agents' wealth, boosting demand for consumption goods. Part of the new spending falls on Home goods, which raises Home output. The other part of the new consumption is spent on Foreign goods, but because prices (wages) in the foreign country are fixed in local (foreign) currency, the increased demand for Foreign goods generates a real appreciation for the Foreign country and leads to an expenditure-switching effect away from Foreign goods.

”$M\uparrow$ *causes* $C\uparrow$. *But does money increases in real or nominal terms*?”

Question: why the exchange rate depreciation doesn’t cause a fall of real money balances at Home? The nominal exchange rate moves with money supply but since P_H and P_F^* are fixed, the Home currency depreciation only raises the Home CPI by a fraction $(1-\gamma)$ of the increase in money supply. Thus, Home agents’ wealth definitely increase in real terms, boosting demand for consumption goods.

$C_H^*\uparrow$

$\nearrow$

$C^*\uparrow$

$\searrow$

$C_F^*\uparrow$ but this effect is dampened by expenditure-switching

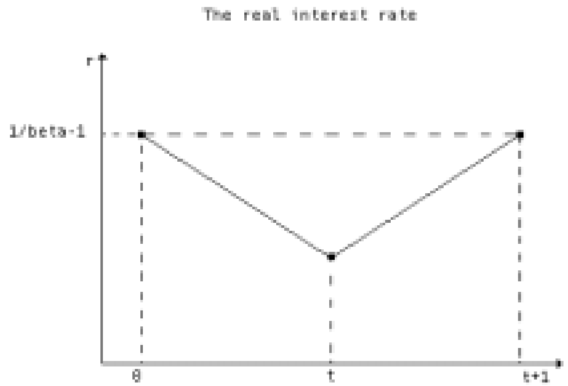
Abroad, the depreciation of the Home currency improves the purchasing power of Foreign incomes, causing an increase in Foreign consumption as well. Part of the increase falls on Home-produced goods. Therefore, because the demand for Home goods increases both at home and abroad, Home output unambiguously increases. However, because of the expenditure-switching effect, the effect on Foreign output is ambiguous: it can be positive or negative, depending on parameter values.

3.2 The behaviour of interest rates and inflation

Real interest rate

The increased demand in the short run puts pressure on prices. As a consequence, in period $t+1$, when nominal rigidities are removed, prices (wages) increase. Because prices increase, in the long run we have a decrease in real incomes and real money balances, causing a fall in consumption expenditure in both countries. Then, by looking at equations (22) and (23), we can understand why the real interest rate falls in the short run: r decreases in the short run to allow consumption to fall in the long run. In other words, the fall in interest rates makes worthwhile to consume more in period t and less in period $t+1$.

Since Home and Foreign agents face the same real interest rates, the fall in consumption must be the same in the two countries.



The above graph represents the path of the real interest rate before the shock (time 0) and after the permanent money shock. The shock occurs at time t and at $t+1$ the economy reaches the steady state.

Technical note: if we assume, for example, that the nominal yield i_{t-1} is "decided" with contracts written at time $t-1$, but it is paid at date t, then the Fisher equations (28) and (29) show that at time 0 there are two "real interest rates", one ex-ante and one ex-post, due to the unexpected change in prices at time t. In the Home country in period 0 the ex-post real interest rate (not shown in the graph) is lower than the ex-ante rate, because of realized inflation (the opposite happens in the Foreign country). The size of the jump from the ex-ante to the ex-post rate depends on the parameters of the model, and for simplicity it is not represented in the graph, which has only one real interest rate at time 0 (the ex-ante, pre-shock steady state level).

Inflation

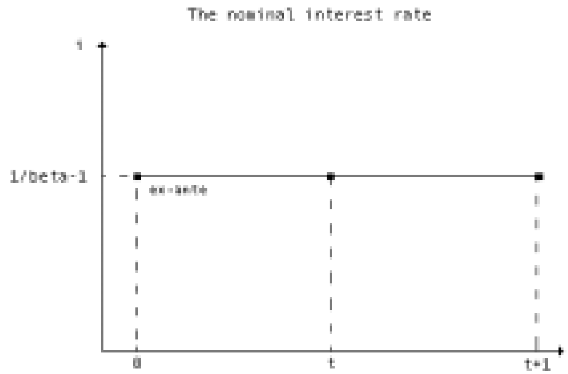
In the short run P_H and P_F^* are fixed and the CPI in both countries is only affected by exchange rate movements. In the short run the Home country has positive inflation and the Foreign country negative inflation, because of the depreciation of the Home currency. From t (short run) to $t+1$ (long run) prices increase in both countries, and inflation rates are the same. Why?

As it is explained later, In CP's model the exchange rate completely adjusts on impact: $E = \overline{E}$. But then the UIP $1 + i = \frac{\overline{E}}{E}(1 + i^*)$ implies $i = i^*$. Therefore, by the Fisher equations (28) and (29): $\frac{\overline{P}}{P} = \frac{\overline{P}^*}{P^*}$. QED

	Inflation	
	from 0 to t	from t to $t+1$
Home	> 0	> 0
Foreign	< 0	> 0

Nominal interest rates

Consider the Home country first. We know that in $t + 1$ the nominal interest rate must be at its steady state level $\frac{1}{\beta} - 1$ (since money is neutral in the long run, as explained later), but what about the short run? By using the equations of the closed form solution (Section 2.2), straightforward calculations show that in period t the nominal interest rate must also be equal to $\frac{1}{\beta} - 1$. Hence, in CP's model the nominal interest rate doesn't change in the short run. This result is entirely dependent on the fact that bond holdings stay the same (this will be explained later), hence the demand for bonds is unaffected by monetary shocks. A change in the demand for bonds would affect the nominal interest rate: an increased demand for bonds would lower the nominal interest rate, a decrease in demand would increase it. Since in CP's model $\Delta B = 0$ after the monetary shock, then $\Delta i = 0$. This feature of CP's model cannot be shared by OR's one. In OR's model, bond holdings are not constant and therefore the nominal interest rate changes.



The result of no change in the nominal interest rate is less surprising than it might appear at first. If all prices were rigid, then a monetary expansion at Home would bring the real interest rate down, and if inflation expectations stay the same or do not

”change as much” as M, the nominal interest rate must fall. The decrease in the nominal interest rate in response to a monetary expansion is called liquidity effect. On the other hand, in models where prices are fully flexible, money growth affects the nominal interest rate one-for-one, while the real is not affected. This is called the Fisher effect. But what happens in CP’s model? In this model prices are fixed for one period, but the exchange rate moves, and so does (to some extent) Home CPI, hence we have both a liquidity and a Fisher effect. moreover, because of CP’s assumptions on the utility function , $\triangle B$ and $\triangle i$ are equal to zero in each period, therefore we can conclude that the liquidity and Fisher effects offset each other, so there are no changes in the nominal interest rate. For an explanation of both the liquidity effect and the Fisher effect, see Romer’s Advanced Macroeconomics, 1st ed., pages 389-95.

3.3 Monetary shocks do not affect the current account

Re-write the current account equations (30) and (31) in the short run and in the long run (starting from and initial $B_0 = 0$ and noting that $B^* = -B$):

short run:

$$B = P_H \frac{Y}{g} - PC \quad ; \quad -\frac{B}{E} = P_F^* \frac{Y^*}{g^*} - P^* C^*$$

long run:

$$\overline{P}\,\overline{C} = \overline{P}_H \frac{\overline{Y}}{\overline{g}} + \delta \overline{B} \quad ; \quad \overline{P}^* \overline{C}^* = \overline{P}_F^* \frac{\overline{Y}^*}{\overline{g}^*} - \delta \frac{\overline{B}}{\overline{E}}$$

where δ is the constant new steady state level of the nominal (equal to the real) interest rate.

Re-write the equilibrium conditions in the goods market (32) and (33) in the short run and in the long run:

short run:

$$\frac{P_H}{P} \frac{Y}{g} = \gamma (C + C^*)$$

$$\frac{P_F^*}{P^*} \frac{Y^*}{g^*} = (1 - \gamma)(C + C^*)$$

long run:

$$\frac{\overline{P}_H}{\overline{P}} \frac{\overline{Y}}{\overline{g}} = \gamma \left(\overline{C} + \overline{C}^* \right)$$

$$\frac{\overline{P}_F^*}{\overline{P}^*} \frac{\overline{Y}^*}{\overline{g}^*} = (1 - \gamma) \left(\overline{C} + \overline{C}^* \right)$$

Hence we can write:

$$\frac{C+B/P}{C^*-B/P} = \frac{\gamma}{1-\gamma} \quad ; \quad \frac{\overline{C}-\delta\overline{B}/\overline{P}}{\overline{C}^*+\delta\overline{B}/\overline{P}} = \frac{\gamma}{1-\gamma}$$

Since equations (22) and (23) imply that the ratio of Home to Foreign consumption is the same in the short and in the long run:

$$\frac{C}{C^*} = \frac{\overline{C}}{\overline{C}^*}$$

and observing that $(1+i)B = (1+\delta)\overline{B}$, we conclude that $B = \overline{B} = 0$.

Intuition: in CP's setup:

$$\frac{Y-G}{Y^*-G^*} = \frac{\gamma}{1-\gamma}\left(\frac{P_H}{EP_F^*}\right)^{-1}$$

→ the elasticity of relative net output demand with respect to relative prices is equal to one, which is also the elasticity of intratemporal substitution in the consumption index. When the short-run relative price of Foreign goods increases, demand for the Home good increases relative to the Foreign good. Home agents' nominal incomes increase relative to Foreign agents but their purchasing power declines proportionally: the real incomes of Home agents stay the same relative to Foreign agents. Therefore, agents have no incentive to lend or borrow internationally to smooth their consumption over time.

Since money shocks do not lead to international redistribution of wealth through current account changes, in CP's model money is neutral in the long run.

3.4 The exchange rate completely adjusts on impact

Assume a permanent monetary shock ($M = \overline{M}$). Consider the Home and Foreign money market equilibrium equations (24) and (25) in the short run and in the long run:

short run:

$$\frac{\overline{M}}{P} = \chi\frac{1+i}{i}C^{\rho} \quad ; \quad \frac{\overline{M}^*}{P^*} = \chi^*\frac{1+i}{1+i-\overline{E}/E}C^{*\rho}$$

long run:

$$\frac{\overline{M}}{\overline{P}} = \chi\frac{1+\delta}{\delta}\overline{C}^{\rho} \quad ; \quad \frac{\overline{M}^*}{\overline{P}^*} = \chi^*\frac{1+\delta}{\delta}\overline{C}^{*\rho}$$

then, by using the PPP condition, the Fisher equations (28) and (29) and the UIP condition (they always hold throughout the model), we get:

$$\frac{E}{\overline{E}} = \frac{(1+r)\overline{P}^{*} - (E/\overline{E})P^{*}}{(1+r)\overline{P}^{*} - P^{*}}$$

that is solved by $E = \overline{E}$.

3.5 The impact of fiscal shocks

An unexpected fiscal expansion has no short-run effect on domestic consumption, but increases domestic demand and employment at unchanged terms of trade. If the shocks are temporary, after one period the economy would move back to the initial equilibrium. When the shock is permanent, the increase in demand for Home goods causes an upward adjustment of wages and prices in the long run. Thus, in the new steady state the relative price of Home goods rises and the Home currency appreciates in real terms. Because of the real wage adjustment, long-run Home output increases by less than public spending, and world consumption falls while prices increase in both countries. The economy reaches an equilibrium corresponding to lower consumption and higher output levels relative to the initial steady state allocation.

4. Welfare analysis

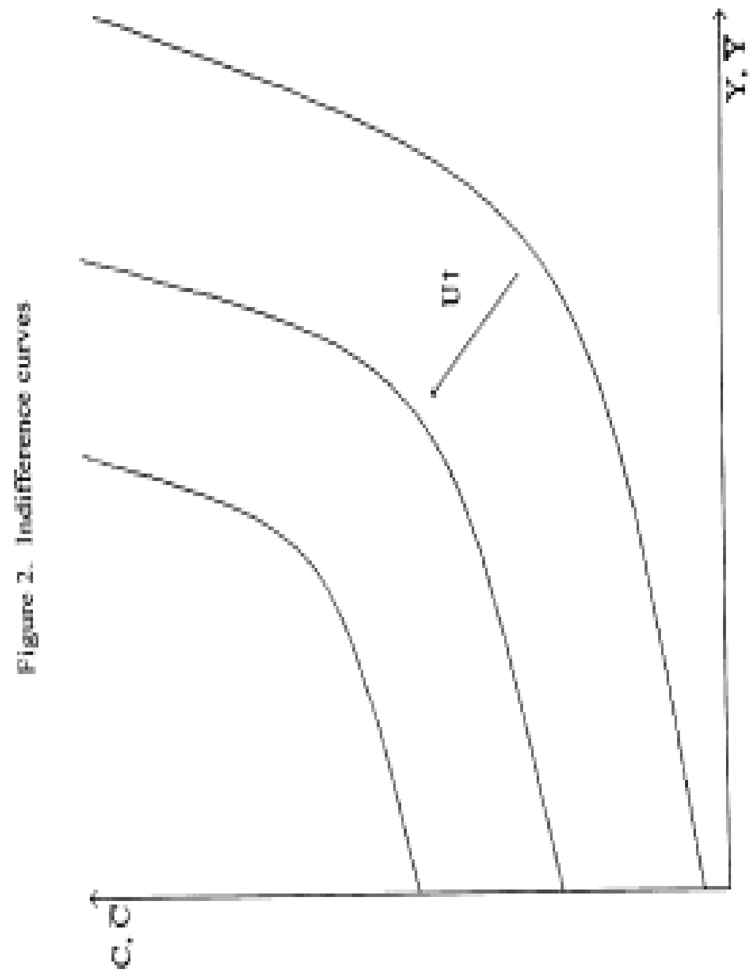
CP's model is a two-country, general equilibrium model, in which agents maximize their lifetime utility. Because of its micro-foundations, it permits the welfare evaluation of international macroeconomic policies.

Welfare analysis can be carried out by looking at the lifetime utility of the Home representative agent. Considering a permanent monetary shock, and remembering that the short run lasts one period only, lifetime utility becomes:

$$U = \frac{C^{1-\rho}}{1-\rho} + \chi \ln \frac{\overline{M}}{P} + V(G) - \frac{\kappa}{2} l^2$$
$$+\beta\left[\frac{\overline{C}^{1-\rho}}{1-\rho} + \chi \ln \frac{\overline{M}}{\overline{P}} + V\left(\overline{G}\right) - \frac{\kappa}{2} \overline{l}^{\,2} \right]$$

Then, the impact on welfare of the agent can be obtained by substituting in the above equation the closed-form expressions of Section 2.2.

Alternatively, the graphical solution of Section 2.3 can be extended to include welfare analysis. To do so, draw the short-run and long-run indifference curves of the representative agent in the YC space, assuming constant utility from real money balances and public goods. After a shock, the GE, LE and ME curves shift, leading the agent to a higher or lower indifference curve. Since utility does not depend only on C and Y, one should also consider the impact on utility of changes in M (liquidity effect) and G, but CP claim that in many cases liquidity effects or changes in the supply of the public good only reinforce the welfare impact of the monetary/fiscal policy shocks.



Source: NBER wp 6307.