# Monopolistic Competition and Price Indexes

Laura Povoledo

15th May 2002

# 1 Types of Demand Functions

Here we will see how to derive the demand functions for all the differentiated goods that form a consumption index.

## 1.1 CES

Each individual in the economy consumes a continuum of differentiated goods, the goods are aggregated according to the following CES consumption index :

$$C_t \equiv \left[\int_0^1 C_t\left(z\right)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}$$

Goods are indexed by $z \in [0,1]$. The demand for each good comes from an expenditure allocation problem. The consumer will allocate optimally his nominal expenditure $P_t C_t$ across each differentiated good, that is, he will solve the following maximization problem:

$$\max_{C_t(z)} \left[ \int_0^1 C_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

$$\text{s.t.} \quad P_t C_t = \int_0^1 p_t(z)\, C_t(z)\, dz$$

Notes:

- In this maximization problem total nominal expenditure $P_t C_t$ and the individual goods' prices are given, but we haven't said yet anything on how to construct a suitable price index $P_t$: this will be done in the following Section. Consider this as the maximization problem faced by an individual who has a given amount of nominal resources to allocate across goods.

- In the above CES consumption index the elasticity of substitution among any two goods $z$ and $z'$ is:

$$-\frac{\vartheta[C_t(z)/C_t(z')]}{\vartheta[p_t(z)/p_t(z')]} \cdot \frac{p_t(z)/p_t(z')}{C_t(z)/C_t(z')} = \theta$$

First order condition of the maximization problem:

$$\left(\frac{C_t(z)}{C_t(z')}\right)^{-\frac{1}{\theta}} = \frac{p_t(z)}{p_t(z')}$$

Or:

$$\frac{C_t(z)}{C_t(z')} = \left(\frac{p_t(z)}{p_t(z')}\right)^{-\theta}$$

Substitute it into the budget constraint:

$$\int_0^1 p_t\left(z\right) C_t\left(z\right) dz = P_t C_t$$

Divide by $C_t\left(z'\right)$:

$\int_0^1 p_t\left(z\right) \frac{C_t(z)}{C_t(z')} dz = \frac{P_t C_t}{C_t(z')}$ for any

$z' \in [0,1]$

Substitute $\frac{C_t(z)}{C_t(z')}$ with $\left(\frac{p_t(z)}{p_t(z')}\right)^{-\theta}$ and simplify:

$$\int_0^1 p_t\left(z\right)\left(\frac{p_t(z)}{p_t(z')}\right)^{-\theta} dz = \frac{P_t C_t}{C_t(z')}$$

$$p_t\left(z'\right)^{\theta} \int_0^1 p_t\left(z\right)^{1-\theta} dz = \frac{P_t C_t}{C_t(z')}$$

$$\int_0^1 p_t\left(z\right)^{1-\theta} dz = \frac{P_t C_t}{C_t(z')} p_t\left(z'\right)^{-\theta} \text{ (*)}$$

Now we make use of the consumption-based price index . The following Section shows that it is written in this way:

$$P_t \equiv \left[\int_0^1 p_t\left(z\right)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}$$

Hence:

$$\int_0^1 p_t\left(z\right)^{1-\theta} dz = P_t^{1-\theta}$$

Then (*) becomes:

$P_t^{1-\theta} = \frac{P_t C_t}{C_t(z')} p_t \left(z'\right)^{-\theta}$

That is:

$P_t^{-\theta} = \frac{C_t}{C_t(z')} p_t \left(z'\right)^{-\theta}$

$C_t \left(z'\right) = \frac{p_t(z')^{-\theta}}{P_t^{-\theta}} C_t$

where we've picked up $z'$ from $[0, 1]$; $z'$ is any good in that interval.

Summing up, the demand for each differentiated good $z \in [0, 1]$ is given by:

$$\boxed{C_t \left(z\right) = \left[\frac{p_t(z)}{P_t}\right]^{-\theta} C_t}$$

We can also observe that

$$\frac{\vartheta C_t(z)}{\vartheta p_t(z)} = -\theta \left[\frac{p_t(z)}{P_t}\right]^{-\theta-1} C_t \cdot \frac{1}{P_t}$$
$$= -\theta \frac{C_t(z)}{p_t(z)} < 0$$

Properties of the CES consumption index:

- The demand function is decreasing in price: $\frac{\vartheta C_t(z)}{\vartheta p_t(z)} < 0$.
- The demand function depends on the relative price $\frac{p_t(z)}{P_t}$: if prices of all goods increase but the relative price stays the same demand doesn't change.
- If $\theta \to 1$ then the CES becomes a Cobb Douglas.

### 1.1.1 A quicker method:

The demand for differentiated goods may also be found as the solution of an expenditure allocation problem: given prices and a desired level of the consumption aggregate $C_t$, the consumer has to decide how to allocate his expenditure across all the different varieties of the goods.

That is, the consumer solves the following maximization problem:

$$\max_{C_t(z)} \; P_t C_t - \int_0^1 p_t(z) \, C_t(z) \, dz$$

$$\text{s.t.} \quad C_t = \left[ \int_0^1 C_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

First-order condition:

$$P_t \frac{\vartheta C_t}{\vartheta C_t(z)} = p_t(z)$$

Since $\frac{\vartheta C_t}{\vartheta C_t(z)} = \left( \frac{C_t(z)}{C_t} \right)^{-\frac{1}{\theta}}$

$$P_t \left( \frac{C_t(z)}{C_t} \right)^{-\frac{1}{\theta}} = p_t(z)$$

Re-arranging:

$$C_t(z) = \left[ \frac{p_t(z)}{P_t} \right]^{-\theta} C_t$$

## 1.2 Cobb Douglas

Consider the problem of a consumer who has to allocate his expenditure between two goods, good $H$ and good $F$. Define the basket of differentiated goods as follows:

$$C_t \equiv \frac{\left(C_t^H\right)^n \left(C_t^F\right)^{1-n}}{n^n (1-n)^{1-n}}$$

The consumer solves the following maximization problem:

$$\max_{C_t^H, C_t^F} \frac{\left(C_t^H\right)^n \left(C_t^F\right)^{1-n}}{n^n (1-n)^{1-n}}$$

$$\text{s.t.} \quad P_t C_t = P_t^H C_t^H + P_t^F C_t^F$$

Note:

- In the above Cobb Douglas consumption index the elasticity of substitution is:

$$-\frac{\vartheta\left[C_t^H / C_t^F\right]}{\vartheta\left[P_t^H / P_t^F\right]} \cdot \frac{P_t^H / P_t^F}{C_t^H / C_t^F} = 1$$

First order condition:

$$\frac{n}{1-n}\left(\frac{C_t^H}{C_t^F}\right)^{-1} = \frac{P_t^H}{P_t^F}$$

Or:
$\frac{C_t^F}{C_t^H} = \frac{1-n}{n} \cdot \frac{P_t^H}{P_t^F}$

Substitute it into the budget constraint:

$$P_t^H C_t^H + P_t^F C_t^F = P_t C_t$$

Divide by $C_t^H$ and simplify:

$$P_t^H + P_t^F \frac{C_t^F}{C_t^H} = \frac{P_t C_t}{C_t^H}$$

Now substitute $\frac{C_t^F}{C_t^H}$ with $\frac{1-n}{n} \cdot \frac{P_t^H}{P_t^F}$:

$$P_t^H + P_t^F \cdot \frac{1-n}{n} \cdot \frac{P_t^H}{P_t^F} = \frac{P_t C_t}{C_t^H}$$

Multiplying by $n$:

$$n \cdot P_t^H + (1-n) \cdot P_t^H = n \cdot \frac{P_t C_t}{C_t^H}$$

$$P_t^H = n \cdot \frac{P_t C_t}{C_t^H}$$

Hence the demand for good $H$ is:

$$\boxed{C_t^H = n \cdot \frac{P_t C_t}{P_t^H}}$$

Analogously, the demand for good $F$ is:

$$\boxed{C_t^F = (1-n) \cdot \frac{P_t C_t}{P_t^F}}$$

Properties of the Cobb Douglas consumption index:

- The demand for each good is a decreasing function of its own price: $\frac{\vartheta C_t^H}{\vartheta P_t^H}$ and $\frac{\vartheta C_t^F}{\vartheta P_t^F}$ are $< 0$.
- The consumer allocates a constant share of its nominal expenditure to each good: $\frac{P_t^H C_t^H}{P_t C_t} = n$ and $\frac{P_t^F C_t^F}{P_t C_t} = (1-n)$.

### 1.2.1 Cobb-Douglas consumption index in two-country models

Call $C_t^H$ consumption (at Home) of the Home good and $C_t^F$ consumption (at Home) of the Foreign good. $C_t \equiv \frac{\left(C_t^H\right)^n \left(C_t^F\right)^{1-n}}{n^n (1-n)^{1-n}}$ is the Home consumption index (an analogous index can be constructed for the Foreign country), and $n$ and $1-n$ are the size of, respectively, the Home and Foreign countries.

Corsetti and Pesenti (QJE 2001) and Obstfeld and Rogoff (NBER wp # 6694, 1998) show that, using this sort of preferences, Home and Foreign per capita consumption levels are always equal. Exact equality follows because the weights in the utility function are the same as the country size. More generally, if they were different, consumption levels would be proportional but not equal. This property then implies that current accounts are always zero in an intertemporal version of the model (starting from a ”no debt” steady state), a useful result if we want to solve the model by taking approximations around the steady state.

# 2 Price Indexes

## 2.1 CES

The consumption-based price index $P_t$ tells how much real consumption $C_t$ the consumer derives from a given nominal expenditure. It measures the least expenditure of differentiated goods that buys one unit of the consumption index. It is found as the solution of the following minimization problem:

$$\min_{C_t(z)} \; P_t C_t = \int_0^1 p_t(z)\, C_t(z)\, dz$$

$$\text{s.t.} \quad \left[\int_0^1 C_t(z)^{\frac{\theta-1}{\theta}}\, dz\right]^{\frac{\theta}{\theta-1}} = 1$$

First order condition:

$$\left(\frac{C_t(z)}{C_t(z')}\right)^{-\frac{1}{\theta}} = \frac{p_t(z)}{p_t(z')}$$

Or:

$$\frac{C_t(z)}{C_t(z')} = \left(\frac{p_t(z)}{p_t(z')}\right)^{-\theta}$$

Re-write the budget constraint in this way:

$$\int_0^1 p_t(z)\, C_t(z)\, dz = P_t \cdot \left[\int_0^1 C_t(z)^{\frac{\theta-1}{\theta}}\, dz\right]^{\frac{\theta}{\theta-1}}$$

And divide by $C_t(z')$:

$$\int_0^1 p_t(z)\frac{C_t(z)}{C_t(z')}dz = P_t\cdot\left[\int_0^1 \left(\frac{C_t(z)}{C_t(z')}\right)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}$$

Substitute $\frac{C_t(z)}{C_t(z')}$ with $\left(\frac{p_t(z)}{p_t(z')}\right)^{-\theta}$:

$$\int_0^1 p_t(z)\left(\frac{p_t(z)}{p_t(z')}\right)^{-\theta} dz =$$

$$= P_t\cdot\left[\int_0^1 \left(\frac{p_t(z)}{p_t(z')}\right)^{1-\theta} dz\right]^{\frac{\theta}{\theta-1}}$$

Collect $p_t(z')$ and simplify:

$$p_t(z')^{\theta}\cdot\int_0^1 p_t(z)^{1-\theta}\, dz =$$

$$= P_t\cdot\left[p_t(z')^{\theta-1}\cdot\int_0^1 p_t(z)^{1-\theta}\, dz\right]^{\frac{\theta}{\theta-1}}$$

$$p_t(z')^{\theta}\cdot\int_0^1 p_t(z)^{1-\theta}\, dz =$$

$$= p_t(z')^{\theta}\cdot P_t\cdot\left[\int_0^1 p_t(z)^{1-\theta}\, dz\right]^{\frac{\theta}{\theta-1}}$$

$$\int_0^1 p_t(z)^{1-\theta}\, dz = P_t\cdot\left[\int_0^1 p_t(z)^{1-\theta}\, dz\right]^{\frac{\theta}{\theta-1}}$$

$$P_t = \left[\int_0^1 p_t(z)^{1-\theta}\, dz\right] \cdot \left[\int_0^1 p_t(z)^{1-\theta}\, dz\right]^{-\frac{\theta}{\theta-1}} =$$

$$= \left[\int_0^1 p_t(z)^{1-\theta}\, dz\right]^{1-\frac{\theta}{\theta-1}}$$

Finally:

$$\boxed{P_t = \left[\int_0^1 p_t(z)^{1-\theta}\, dz\right]^{\frac{1}{1-\theta}}}$$

## 2.2 Cobb Douglas

The consumption-based price index $P_t$ solves the following minimization problem:

$$\min_{C_t^H, C_t^F} \; P_t C_t = P_t^H C_t^H + P_t^F C_t^F$$

$$\text{s.t.} \qquad \frac{\left(C_t^H\right)^n \left(C_t^F\right)^{1-n}}{n^n (1-n)^{1-n}} = 1$$

First order condition:

$$\frac{n}{1-n}\left(\frac{C_t^H}{C_t^F}\right)^{-1} = \frac{P_t^H}{P_t^F}$$

Or:

$$\frac{C_t^F}{C_t^H} = \frac{1-n}{n} \cdot \frac{P_t^H}{P_t^F}$$

Re-write the budget constraint as follows:

$$P_t^H C_t^H + P_t^F C_t^F = P_t \frac{\left(C_t^H\right)^n \left(C_t^F\right)^{1-n}}{n^n (1-n)^{1-n}}$$

Divide it by $C_t^H$ and simplify:

$$P_t^H + P_t^F \frac{C_t^F}{C_t^H} = P_t \frac{\left(C_t^H\right)^{n-1} \left(C_t^F\right)^{1-n}}{n^n (1-n)^{1-n}}$$

$$P_t^H + P_t^F \cdot \frac{C_t^F}{C_t^H} = \frac{1}{n^n (1-n)^{1-n}} \cdot P_t \cdot \left(\frac{C_t^F}{C_t^H}\right)^{1-n}$$

Substitute $\frac{C_t^F}{C_t^H}$ with $\frac{1-n}{n} \cdot \frac{P_t^H}{P_t^F}$ and simplify:

$$P_t^H + P_t^F \cdot \left(\frac{1-n}{n} \cdot \frac{P_t^H}{P_t^F}\right) =$$

$$= \frac{1}{n^n (1-n)^{1-n}} \cdot P_t \cdot \left(\frac{1-n}{n} \cdot \frac{P_t^H}{P_t^F}\right)^{1-n}$$

$$P_t^H + \frac{1-n}{n} \cdot P_t^H = \frac{n^{n-1}(1-n)^{1-n}}{n^n(1-n)^{1-n}} \cdot P_t \cdot \left(\frac{P_t^H}{P_t^F}\right)^{1-n}$$

Multiply by $n$ and simplify:

$$n \cdot P_t^H + (1-n) \cdot P_t^H = P_t \cdot \left(\frac{P_t^H}{P_t^F}\right)^{1-n}$$

$$P_t = \left(\frac{P_t^H}{P_t^F}\right)^{n-1} \cdot P_t^H$$

Hence the price index is:

$$\boxed{P_t = \left(P_t^H\right)^n \left(P_t^F\right)^{1-n}}$$

# 3 Monopolistic Competition

Now that we know how to derive the demand functions from a given consumption index, we can look at the role it plays in the sticky price monetary models. In all that follows assume, for simplicity, a closed economy.

## 3.1 In the goods market

As in Pierpaolo Benigno, 2000.

The economy is composed of a continuum of infinitely-lived individuals, whose total is normalized to unity. Each individual consumes a basket of differentiated goods, but supplies only one good. As a result, each individual enjoys a monopoly power over the good he produces.

The consumption index is:

$$C_t \equiv \left[\int_0^1 c_t\left(z\right)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}$$

Adding government, demand

functions for the differentiated goods are:

$$y_t(z) = \left[\frac{p_t(z)}{P_t}\right]^{-\theta} (C_t + G_t)$$

(Assuming that the government allocates its total expenditure $G_t$ on differentiated goods in the same fashion as individuals, i.e. $g_t(z) = \left[\frac{p_t(z)}{P_t}\right]^{-\theta} G_t$. Then $y_t(z) = c_t(z) + g_t(z)$ to find the above).

The representative household solves the problem:

$$\max E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[ \begin{array}{c} \frac{C_{t+j}^{1-\sigma}}{1-\sigma} + \frac{\chi}{1-\varepsilon} \left( \frac{M_{t+j}}{P_{t+j}} \right)^{1-\varepsilon} \\ -\frac{\kappa}{2} \left( y_{t+j}(z) \right)^2 \end{array} \right] \right\}$$

s.t. $B_t P_t + M_t = (1 + r_{t-1}) B_{t-1} P_t +$
$+ M_{t-1} + TR_t + p_t(z) y_t(z) - P_t C_t$

$$y_t(z) = \left[\frac{p_t(z)}{P_t}\right]^{-\theta} (C_t + G_t)$$

First order conditions:

1. $C_t^{-\sigma} = \beta E_t \left[(1 + r_t) C_{t+1}^{-\sigma}\right]$
2. $\chi \left(\frac{M_t}{P_t}\right)^{-\varepsilon} = C_t^{-\sigma} \left(\frac{i_t}{1+i_t}\right)$
3. $p_t(z) = \frac{\theta}{\theta-1} \frac{\kappa y_t(z)}{\frac{C_t^{-\sigma}}{P_t}}$

To characterize the behaviour of the economy, we need the above first order conditions, plus:

The budget constraint:

4. $B_t P_t + M_t = (1 + r_{t-1}) B_{t-1} P_t +$
$M_{t-1} +$
$+ TR_t + p_t(z) y_t(z) - P_t C_t$

The demand function:

5. $y_t(z) = \left[\frac{p_t(z)}{P_t}\right]^{-\theta}(C_t + G_t)$

The government budget constraint:

6. $\frac{M_t - M_{t-1}}{P_t} = G_t + TR_t$

The price index:

7. $P_t = \left[\int_0^1 p_t(z)^{1-\theta}\, dz\right]^{\frac{1}{1-\theta}}$

The Fisher parity condition:

8. $1 + i_t = E_t \frac{P_{t+1}}{P_t}(1 + r_t)$

### 3.1.1 Symmetric competitive equilibrium:

$p_t(z) = P_t$

$y_t(z) = Y_t$

$Y_t = C_t + G_t$

$B_t = 0$: since every individual is the same, there is no scope for one individual lending to another one

### 3.1.2 Steady state:

In a steady state where $\overline{G} = 0$ (steady state variables are indexed by upperbars) we have that:

$\overline{Y} = \overline{C}$

$\overline{Y}_t^{-\sigma} = \frac{\theta}{\theta-1}\kappa\overline{Y}$

$\overline{Y} = \left(\frac{\theta-1}{\theta\kappa}\right)^{\frac{1}{1+\sigma}}$

Now we can make the following remarks:

**Remark 1**: With monopolistic competition, the steady state level of output is lower than with perfect competition, hence it is inefficient. With perfect competition $\overline{Y}_{pf} = \left(\frac{1}{\kappa}\right)^{\frac{1}{1+\sigma}}$ is the steady state level of output (to show this, re-do the maximization problem assuming that producers take prices as given).

**Remark 2**: The inefficiency arises because prices are above the marginal cost. In fact, the first order condition for production: $p_t(z) = \frac{\theta}{\theta-1}\frac{\kappa y_t(z)}{\frac{C_t^{-\sigma}}{P_t}}$ implies $p_t(z) > MC$, since $\frac{\theta}{\theta-1} > 1$ and $MC = \frac{\kappa y_t(z)}{\frac{C_t^{-\sigma}}{P_t}}$.

**Why?**: $\kappa y_t(z) = \frac{\vartheta U}{\vartheta y_t(z)}$ is the marginal cost in utility units of producing one extra unit of output, $\frac{C_t^{-\sigma}}{P_t} = \frac{\vartheta U}{\vartheta(P_t C_t)}$ is the marginal benefit of having one extra unit of nominal income. Hence $\frac{\kappa y_t(z)}{\frac{C_t^{-\sigma}}{P_t}}$ is the nominal marginal cost of production, in monetary units.

**Remark 3**: The monopolistic competition distortion can be eliminated with a proportional tax on income. Assume that $M_t - M_{t-1} + \tau \cdot p_t(z) y_t(z) = P_t G_t + P_t TR_t$ is the government budget constraint, where $\tau$ is the proportional rate of taxation. If $\tau = 1 - \left(\frac{\theta}{1-\theta}\right)^{\frac{1}{\sigma}}$ then the steady state level of output is always efficient.

### 3.1.3 Sticky prices

Assume now that prices are not fully flexible, but fixed for one period, or adjusted with some positive probability, as in the Calvo pricing. With sticky prices, the first-order condition for production: $p_t(z) = \frac{\theta}{\theta-1}\frac{\kappa y_t(z)}{\frac{C_t^{-\sigma}}{P_t}}$ doesn't hold. Hence there is another inefficiency in the economy, coming from sticky prices, because agents are not optimizing. Prices are assumed to be sticky in the short run, but flexible in the long run, and there are some more remarks:

**Remark 4**: With monopolistic competition, when prices are fixed firms are willing to meet the increased demand because prices are above the marginal cost. This is why monopolistic competition is introduced in the models.

**Remark 5**: Monopolistic competition does not influence the dynamics of the model. It affects the dynamics of the Obstfeld and Rogoff's model (JPE 1995) because it determines the allocation of expenditure between Home and Foreign goods since they enter the consumption index in this way:

$$C_t \equiv \left[ \int_0^n c_t(z)^{\frac{\theta-1}{\theta}} dz + \int_n^1 c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

but if they had constructed separate consumption indexes for Home and Foreign goods, for example (as in Pierpaolo Benigno 2000):

$$C_t^H \equiv \left[ \left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n c_t\left(z\right)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

$$C_t^F \equiv \left[ \left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 c_t\left(z\right)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

and aggregated them, for example, as in the Cobb Douglas: $C_t \equiv \frac{\left(C_t^H\right)^n \left(C_t^F\right)^{1-n}}{n^n (1-n)^{1-n}}$, or with a CES sub-utility function, then they would have found that the degree of monopolistic competition is not responsible for the transmission of monetary shocks.

## 3.2 In the labour market

As in Hau, JIE 2000.

Assume that there are monopolistically competitive firms, and individuals are monopolistic suppliers of their own labour effort. Index firms and the goods they produce by $z \in [0,1]$, individuals and their labour efforts by $i \in [0,1]$. We have a CES production function where all individuals' labour supplies are needed:

$$Y_t(z) = \left[\int_0^1 l_t(i)^{\frac{\phi-1}{\phi}}\, di\right]^{\frac{\phi}{\phi-1}}$$

From the firms' profit maximization problem:

$$\max p_t(z)\, y_t(z) - \int_0^1 w_t(i)\, l_t(i)\, di$$

$$\text{s.t.} \quad y_t(z) = \left[\int_0^1 l_t(i)^{\frac{\phi-1}{\phi}}\, di\right]^{\frac{\phi}{\phi-1}}$$

we derive the labour demand for each type of labour:

$$l_t(i) = \left(\frac{w_t(i)}{p_t(z)}\right)^{-\phi} y_t(z)$$

Define the consumption index as

follows:

$$C_t \equiv \left[\int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}$$

The representative household solves the problem:

$$\max E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[ \begin{array}{c} \frac{C_{t+j}^{1-\sigma}}{1-\sigma} + \frac{\chi}{1-\varepsilon}\left(\frac{M_{t+j}}{P_{t+j}}\right)^{1-\varepsilon} \\ -\frac{\kappa}{2}\left(l_{t+j}(i)\right)^2 \end{array} \right] \right\}$$

$$\begin{aligned} \text{s.t. } & B_t P_t + M_t = (1 + r_{t-1}) B_{t-1} P_t + M_{t-1} \\ & + TR_t + w_t(i)\, l_t(i) - P_t C_t + \int_0^1 \pi_t(i,z)\, dz \end{aligned}$$

$$l_t(i) = \left(\frac{w_t(i)}{p_t(z)}\right)^{-\phi} y_t(z)$$

$\pi_t(z)$ are the firm profits:

$$\pi_t(z) = p_t(z)\, y_t(z) - \int_0^1 w_t(i)\, l_t(i)\, di$$

And $\pi_t(i,z)$ is the share held by each individual (for example, everyone holds an equal share). Profits arise because of the monopolistic competition assumption in the goods market.

The rest of the analysis is the same as in the above Section. Corsetti and Pesenti, QJE 2001, and Hau, JIE 2000, show that the international transmission mechanism with wage rigidity is similar to the one with price rigidity.

The advantage of this approach is that we can introduce nominal rigidities by either assuming sticky nominal prices or sticky nominal wages (or both). There are some reasons why you may want to introduce sticky nominal wages in your models:

- Christiano et al. (EER 1997)

have pointed out that firm profits decrease after a monetary contraction, and that this observation is unexplained by a combination of sticky product prices and a flexible factor (labour) market. However, model with sticky wages predicts procyclical firm profits.

- Chari, Kehoe and McGrattan (EJ 2002?) claim that staggering of prices alone do not generate enough persistence of exchange rates in the models, because firms respond to the rise in marginal cost as they try to increase output (with a CES prices are constant mark-up over marginal cost). Wage rigidity may be an additional channel to introduce persistence, by dampening the increase in marginal costs (Erceg 1997). You may also want to have a look at Bergin and Feenstra's paper, where translog preferences are introduced instead, in order to slow down the adjustment of prices after a monetary shock.

# 4 Further reading

An excellent reference: Jae Wan Chung, ”Utility and Production Functions”, Blackwell, Cambridge, Mass., 1994. This book has all the functional forms you can possibly imagine, with all the properties.

Other very useful material can

be found browsing Mark Gertler's webpage:

http://www.econ.nyu.edu/dept/courses/gertler

and Lars Svensson's one:

http://www.princeton.edu/~svensson